

4) Integration of Rational Functions

Rules: $\int \frac{1}{x} dx = \ln|x| + c$
 $\int \frac{1}{ax} dx = \frac{1}{a} \int \frac{1}{x} dx = \frac{1}{a} \cdot \ln|x| + c$
 $\int \frac{1}{f(x)} dx = \frac{1}{f'(x)} \cdot \ln|f(x)| + c$ where $f(x)$ is linear
e.g $\int \frac{1}{ax+b} dx = \frac{1}{a} \cdot \ln|ax+b| + c$

Examples: Integrate these functions

1) $\int \frac{4}{x} dx$

$$\int \frac{4}{x} dx = 4 \cdot \int \frac{1}{x} dx = 4 \cdot \ln|x| + c$$

2) $\int \frac{4}{5x+3} dx$

$$\int \frac{4}{5x+3} dx = 4 \cdot \int \frac{1}{5x+3} dx = 4 \cdot \frac{1}{5} \cdot \ln|5x+3| + c = \frac{4}{5} \ln|5x+3| + c$$

3) $\int \frac{x^2+3x}{x^2} dx$

$$\int \frac{x^2+3x}{x^2} dx = \int \frac{x^2}{x^2} + \frac{3x}{x^2} dx = \int 1 + \frac{3}{x} dx = x + 3 \ln|x| + c$$

4) $\int \frac{3x^2+4x-6}{x} dx$

$$\int \frac{3x^2+4x-6}{x} dx = \int \frac{3x^2}{x} + \frac{4x}{x} - \frac{6}{x} dx = \int 3x + 4 - \frac{6}{x} dx = \frac{3x^2}{2} + 4x - 6 \ln|x| + c$$

5) $\int \frac{7x^3-5x}{2x^2} dx$

$$\int \frac{7x^3-5x}{2x^2} dx = \int \frac{7x^3}{2x^2} - \frac{5x}{2x^2} dx = \int \frac{7}{2}x - \frac{5}{2x} dx = \int \frac{7}{2}x - \frac{5}{2} \cdot \frac{1}{x} dx = \frac{7x^2}{4} - \frac{5}{2} \ln|x| + c$$